Convergence of Double Auctions to Pareto Optimal Allocations in an Edgeworth Box

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Prior Studies of Double Auctions

- Partial equilibrium settings
- Double auctions populated by
  - Profit-motivated human traders
  - “Zero-intelligence” or minimal intelligence traders
- Both converge to approximate Pareto optimal allocations

Present Study

- What happens to allocative efficiency of double auctions in general equilibrium settings under classical assumptions

- We use Edgeworth box exchange economies

- To what extent are the “zero-intelligence” trader results applicable to general equilibrium settings?

An Overview

- In double auctions, very low levels of rationality seems to be sufficient for the markets to approximate competitive allocations under a wide variety of partial equilibrium conditions.

- Gode and Sunder, JPE (1993)

- There exist lower bounds on efficiency (fraction of producer and consumer surplus extracted) of double auctions.

- For many familiar market conditions, these bounds are relatively high.

- Gode and Sunder (1993b)

What Makes Markets Allocationally Efficient?

- Magnitude of efficiency of double auctions is determined by the shape of extra-marginal demand and supply
- Two rules jointly raise the efficiency substantially by reducing the chance of intra-marginal traders being displaced:
  - Buyers and sellers abide by bids/asks
  - Higher bids and lower asks given priority
  - Gode and Sunder (1997)
- Double auctions are more efficient than others (e.g., sealed bid) because more conditions have to be fulfilled for an inefficient transaction to occur in DA
- Auctions that accumulate bids/asks before processing are more efficient in allocation but less efficient in price discovery.

Model Economy

General Equilibrium Setting

- In a two-dimensional Edgeworth box
- Two types of traders can exchange their commodity endowment
- Each has convex preferences
- Examine behavior under human and ZI traders

Model Economy

- Agents have Cobb-Douglas utility
- $U = c_1^{\alpha} c_2^{1-\alpha}$ and $V = c_1^{\beta} c_2^{1-\beta}$
- (x_1, y_1) is the endowment point
- Contract curve: $(1-\alpha)(1-y_1)/\alpha y_1 = (1-\beta)(1-x_1)/\beta x_1$
- CE consumption: $(\alpha(x_1+y_1/p^e), (1-\alpha)(p^e x_1+y_1)$

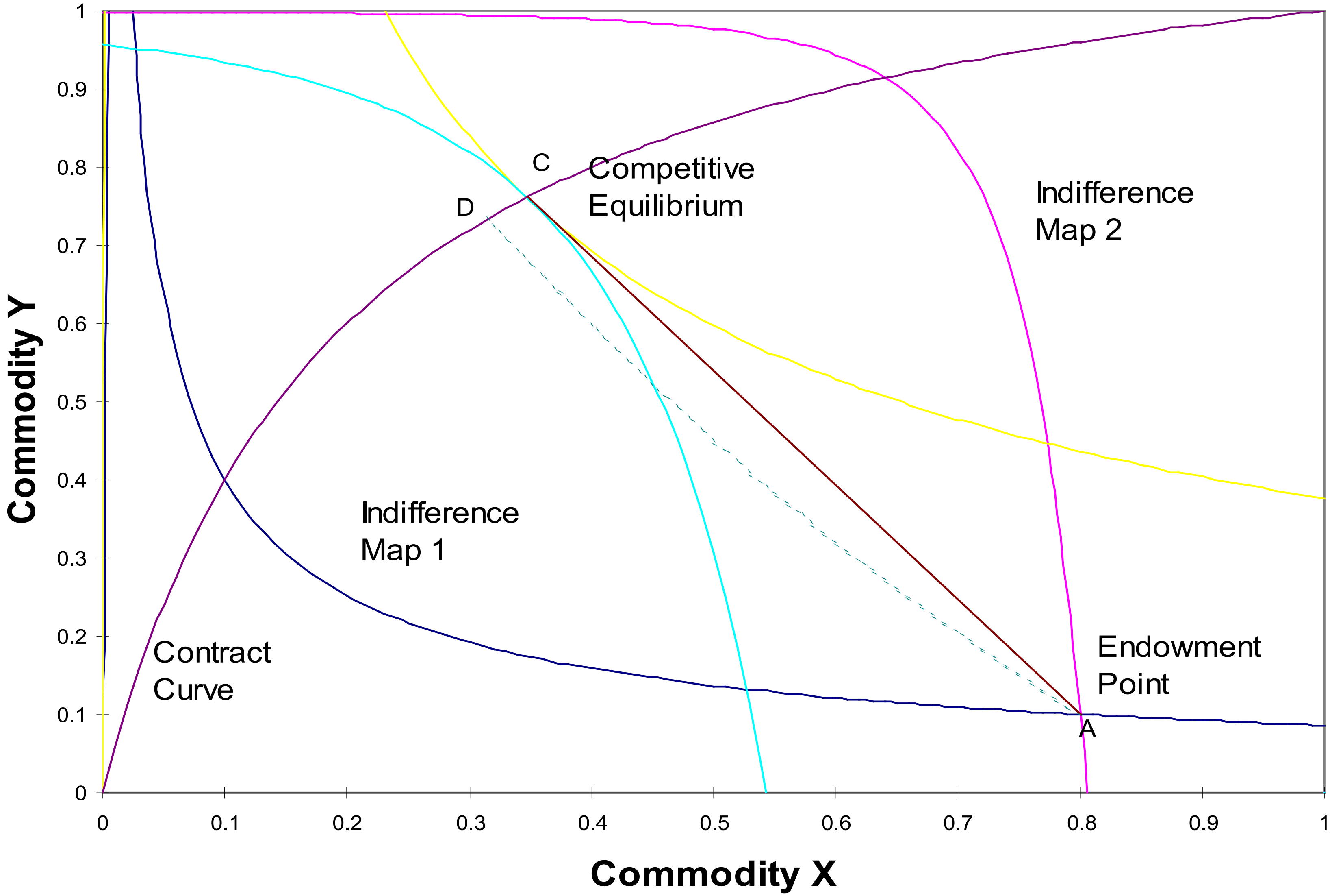
Competitive Equilibrium

- Equilibrium price

  $p^e = ((\alpha y_1 + \beta(1-y_1))/(1-\alpha)x_1+(1-\beta)(1-x_1))$ in units of y per unit of x.

- In Figure 1
  - A is the endowment point,
  - C is the competitive equilibrium consumption bundle,
  - Slope of line AC is the CE price p^e.

Competitive Equilibrium



Human Traders

Human Subject Market

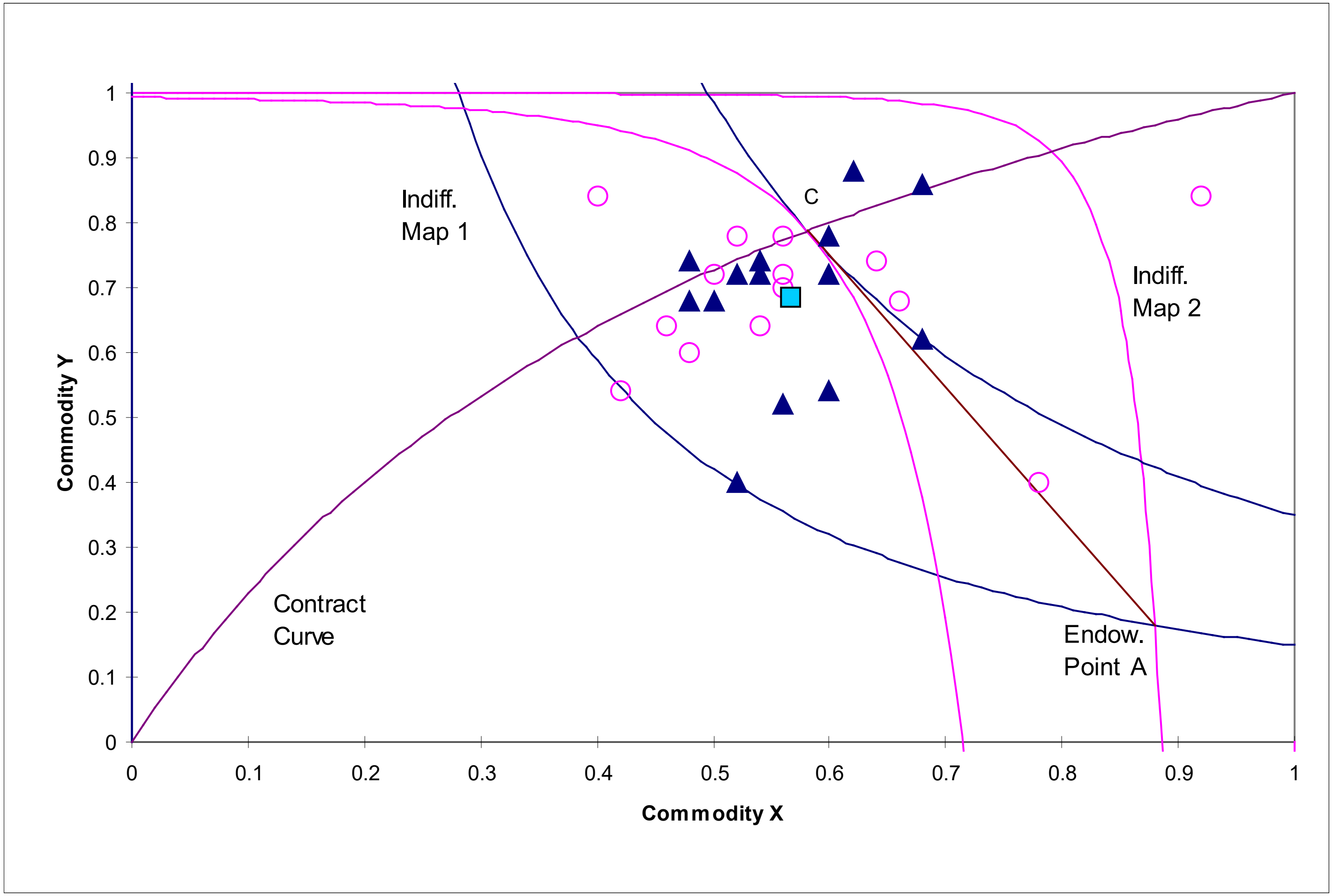
- Two groups of 14 subjects each
- Group 1 members: 44 Green, 9 Red Chips
- $U = c_1^{.6} c_2^{.4}$
- Group 2 members: 6 green, 41 red chips,
- $V = c_1^{.8} c_2^{.2}$
- Subjects free to trade (exchange of integer numbers of green and red chips)

Competitive Equilibrium in Human Subject Market

- Competitive equilibrium quantities
  - Group 1: 29 green, 39 red chips
  - Group 2: 21 green, 11 red chips
- Competitive equilibrium price: 0.49 green chips per red chip
- Competitive equilibrium utility for Group 1 rises from 0.93 to 1.31 and Group 2 utility rises from 0.35 to 0.73
- The contract curve: Red = 8 Green /(3 + 5 Green).

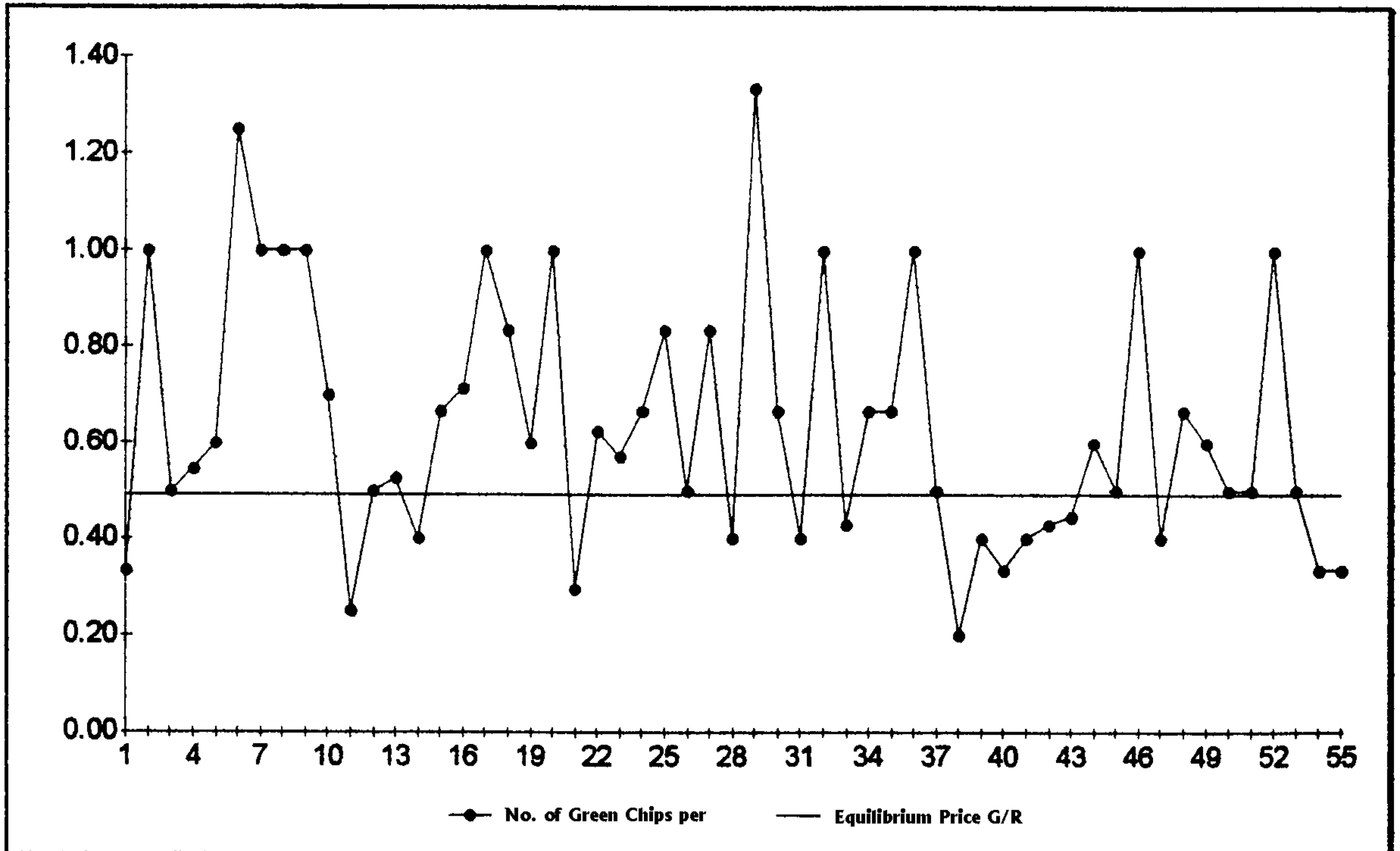
Results in Figure 2

- Solid triangles are Group 1 final holdings [Hollow circles for Group 2]
- Average holdings of the two groups: solid blue square (0.57,0.69).
- Group 1 utility rose from 0.93 to 1.22 (93% of max utility of 1.31)
- Group 2 utility rose from 0.35 to 0.80 (110% of max utility of 0.73)



Prices in Human Market

- Prices stated as the number of green chips exchanged for a red chip
- All transactions were for integer number of chips
- Competitive equilibrium price: 0.49; Average transaction price in market: 0.64



Human Market

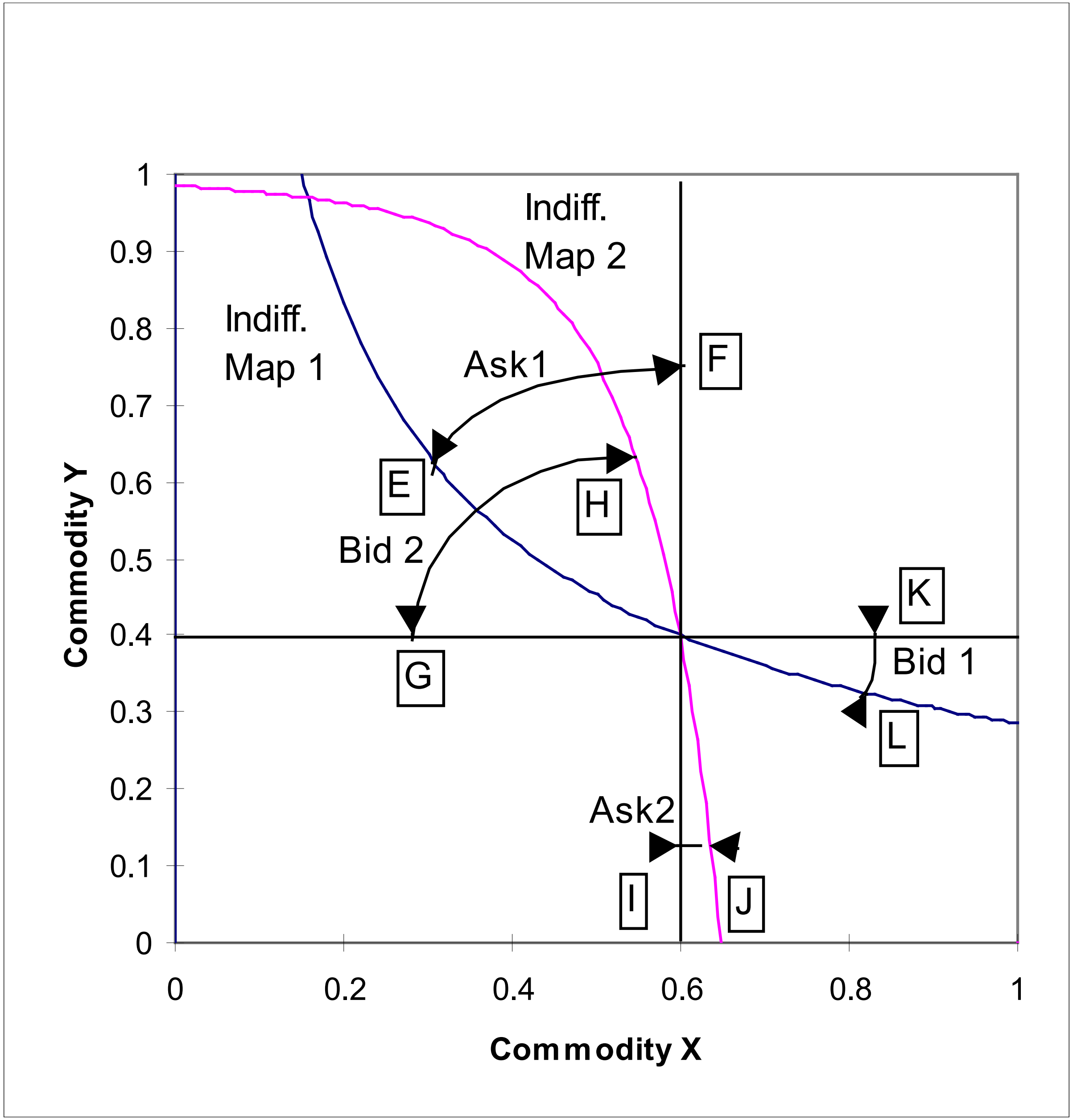
- Subjects got close to the contract curve and competitive equilibrium
- Fell short on utility maximization
- Single shot experiment
  - With experience, better understanding of instructions, and fewer restrictions on trading in integers, human performance may improve

Zero (Minimally) Intelligent Traders

ZI Traders

- Agent computes slope of indifference map. Suppose slope is r radians (expressed so the slope of the tangent is the number of units of y one is willing to sacrifice in order to gain one unit of x, $0 \leq r \leq \pi/2$)
- Trader picks two random numbers
- One distributed uniformly over the range 0-r. The second over the range r-π/2.
- The trader offers the first random number as the price at which it is willing to buy commodity x in exchange for y
- It offers the second random number as the price at which it is willing to sell x in exchange for y.
- That is, the first random number is its bid for x, and the second its ask for x (see Fig. 4)

Figure 4: Zero-Intelligence Bidding in Edgeworth Box



Transaction Unit Size

- In simulation, each transaction should have a finite size to finish in finite time

- In simulation of discrete steps of finite size, curvature of the indifference maps should be accounted for.

- At finite distance, tangent deviates from the indifference map.

- Angles for bids and asks in Figure 4 modified
  - Trader 1: bid angles KL and ask angles EF
  - Trader 2: bid angle GH and ask angles IJ

- To retain symmetry with respect to the two commodities

  - Specify size as length of the radius a, so $a = (x^2 + y^2)^{1/2}$,

  - x and y are the quantities

Minimal Rationality

- The bidding rule assumes minimal rationality
- Agents strive to climb their utility hill through locally beneficial exchanges
- Have no knowledge or awareness of their environment to optimize the bids and offers
- Choose randomly from the available set of steps that will raise them to a higher plane
- Act locally, do not optimize

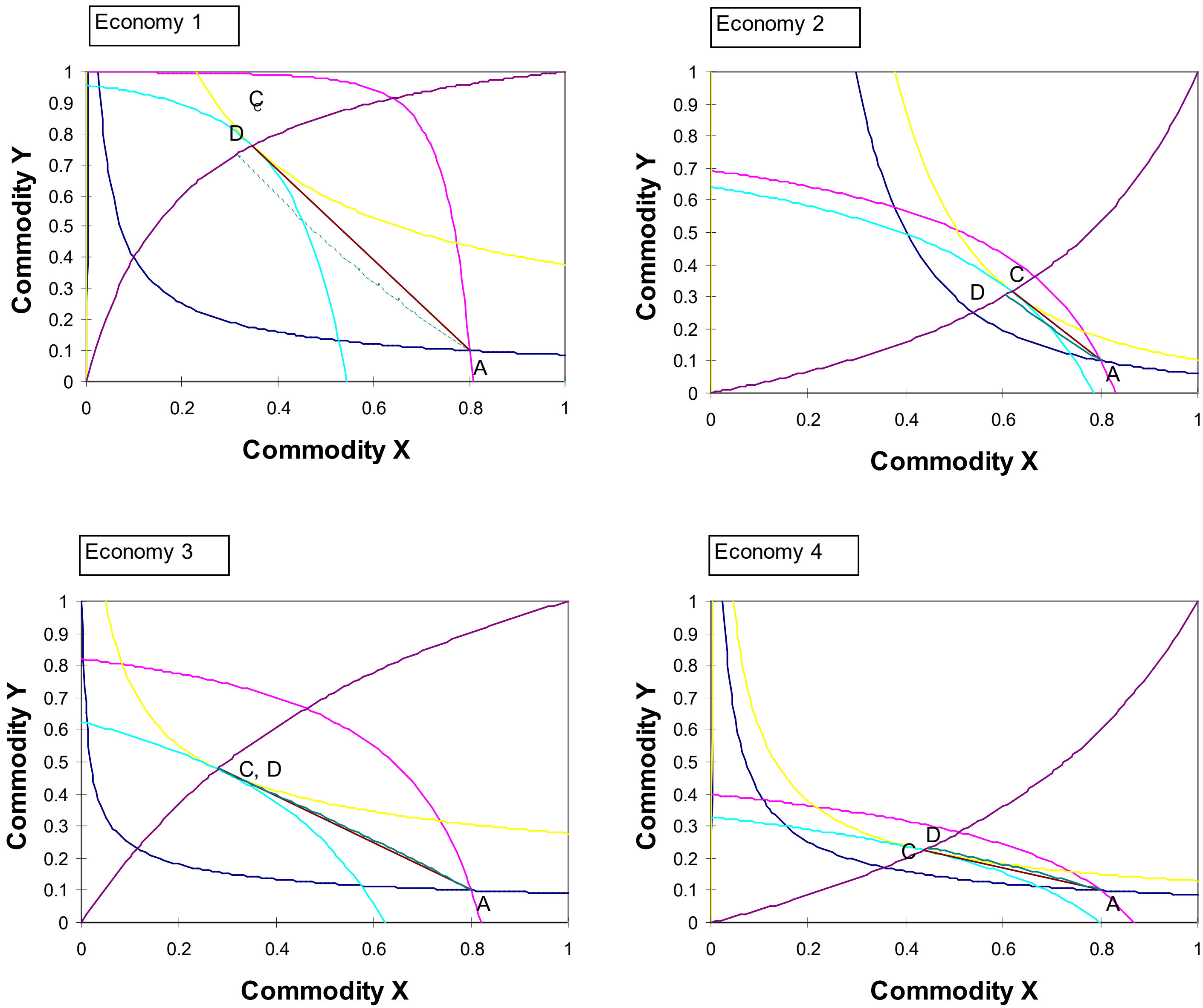
ZI Computational Economies

- Table 2 shows the parameters of four computational economies for which we present the results.
- Indifference maps, competitive equilibria and CE prices for the four economies are shown graphically in Figure 5.
- A is initial endowment point, C is CE.

Table 2: Parameters of Computational Economies

Parameter	Economy 1	Economy 2	Economy 3	Economy 4
α: Trader 1	0.4	0.7	0.3	0.4
β: Trader 2	0.8	0.4	0.5	0.2
Initial Endowment: (x, y)	(0.8, 0.1)	(0.8, 0.1)	(0.8, 0.1)	(0.8, 0.1)
Number of Iterations	2,000	2,000	2,000	2,000
Discrete Transaction Size	0.02	0.02	0.02	0.02

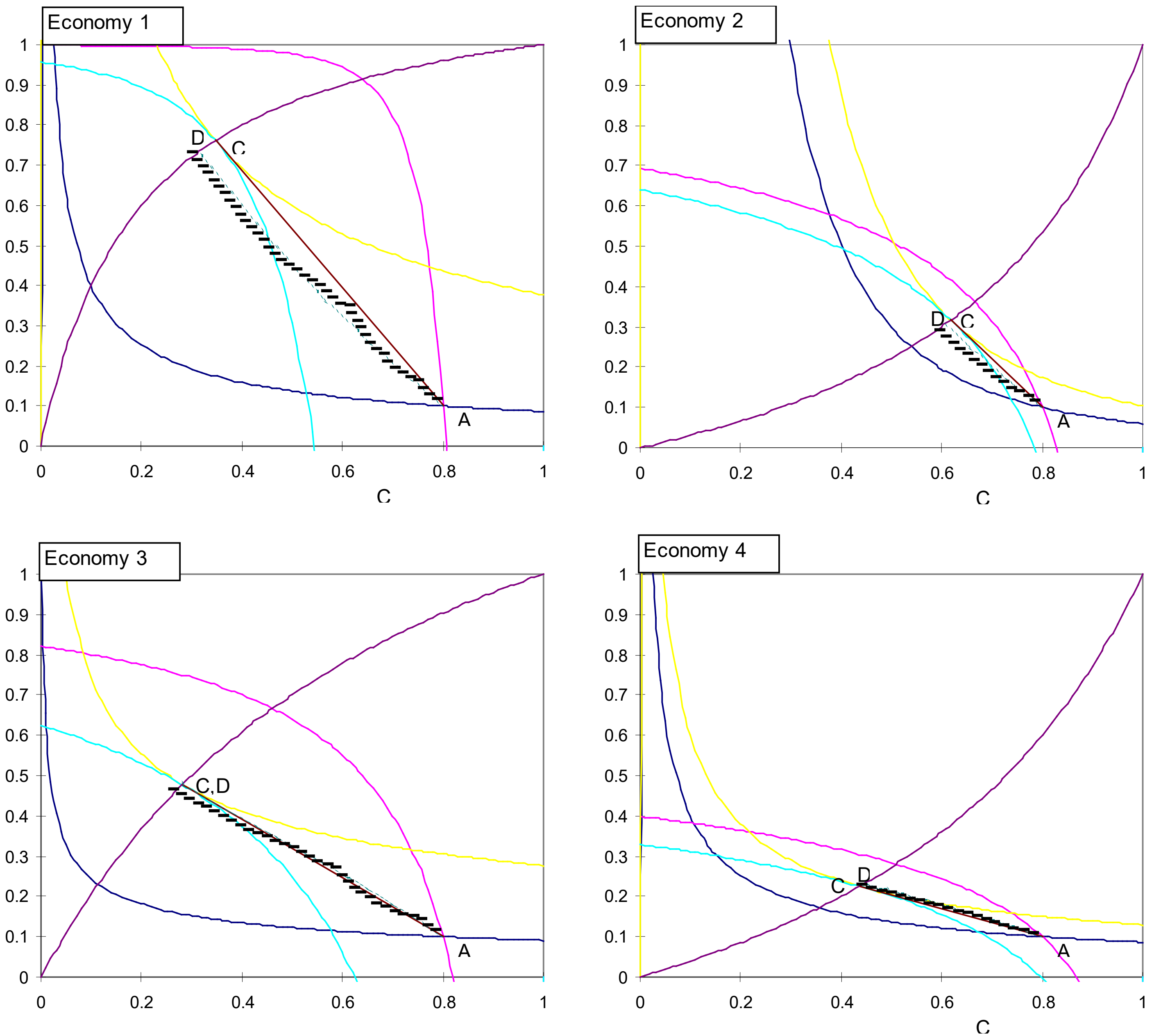
Fig. 5: Four Computational Economies



Convergence to Contract Curve

- Path of consumption bundles starting from A to the end in short bars
- All economies terminate at the contract curve
- Some small deviations due to discreteness of simulation
- Can be reduced by reducing size of steps

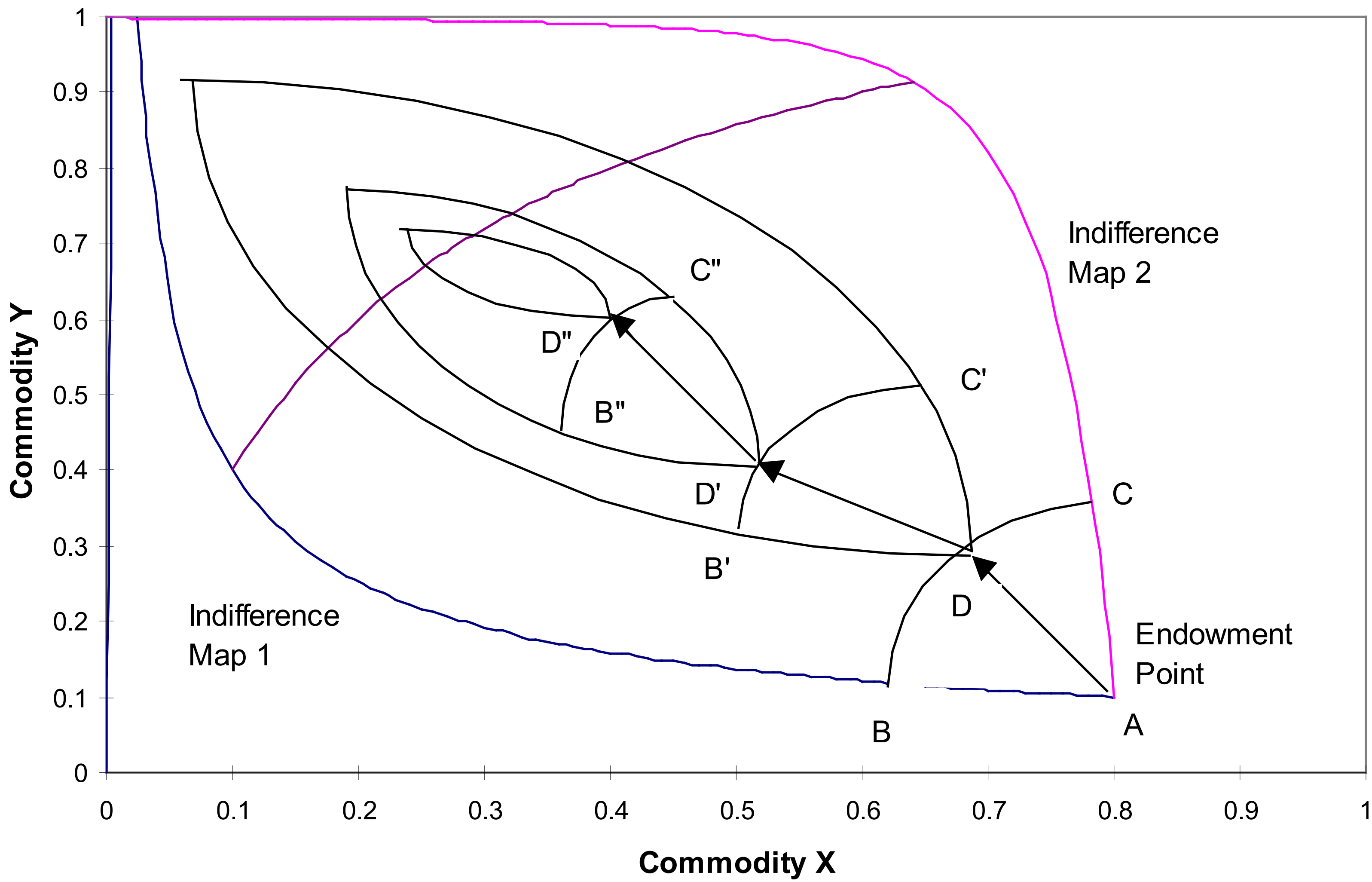
Endowment Paths (Initial-Final)



Why Do Paths End at Contract Curve?

- Figure 7: A is arbitrarily initial endowment
- A is the center of arc BC and simulation step size a is its radius.
- Points on arc BC are feasible for the traders' consumption after first transaction.
- The double auction mechanism guarantees that one of these points will be chosen
- Same argument continues to next steps

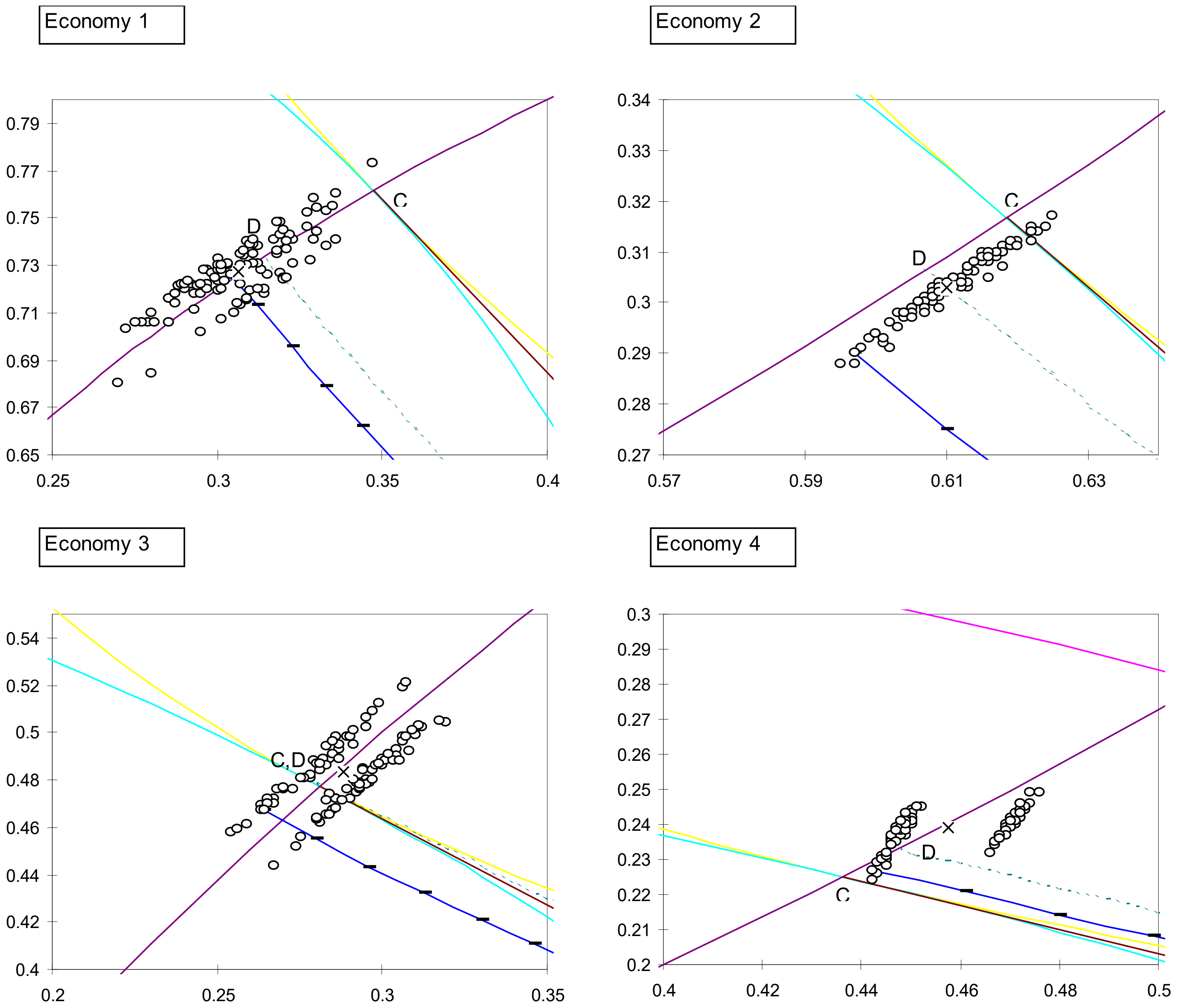
Movement to the Contract Curve



Competitive Equilibrium

- Do the economies converge to CE?
- Figure 8: termination points of 100 runs of the four economies
- Cross is the mean or centroid of 100 ends, is not close to C, the CE
- Ends distributed around D, the intersection of contract curve and locus of the curve that bisects the angle between indifference maps

Distribution of Path End-Points

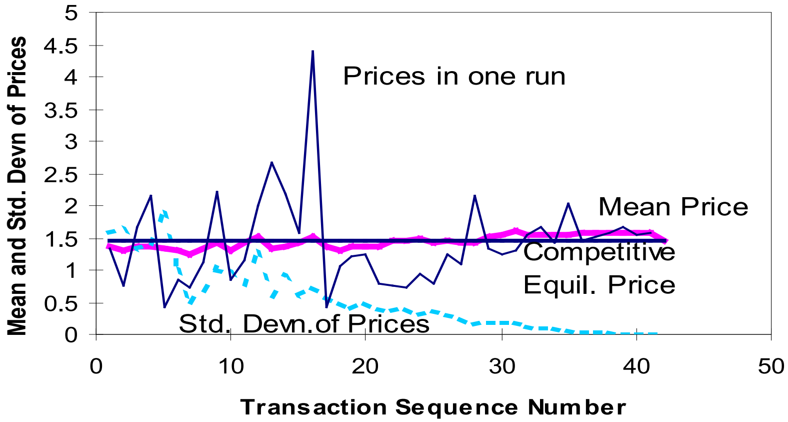


Transaction Prices

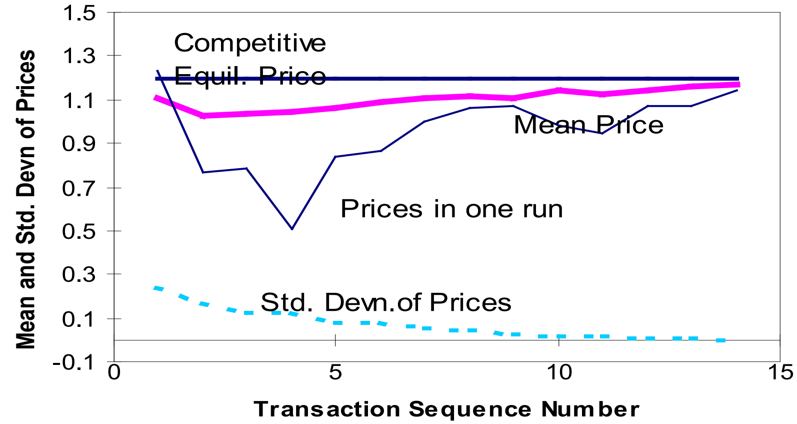
- Figure 9: transaction price series for a single run of each of four economies
- Average and standard deviation of the i^{th} transaction for each economy vs. CE price
- Transaction prices tend to follow the bisection locus.
- Prices may converge to CE price, yet allocations may diverge

Transaction Prices

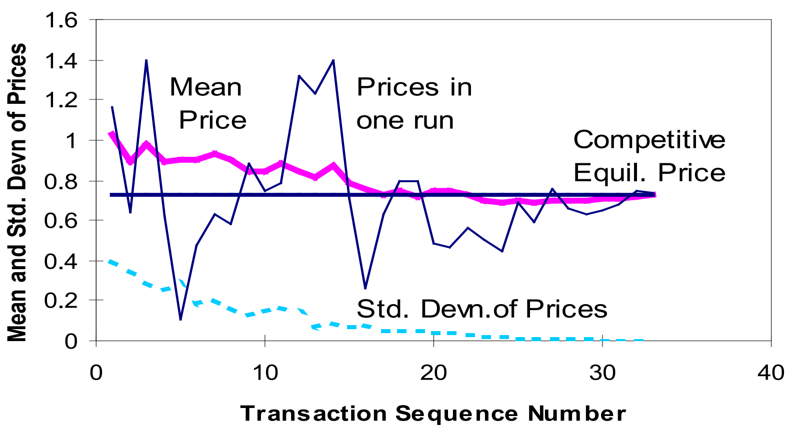
Economy 1



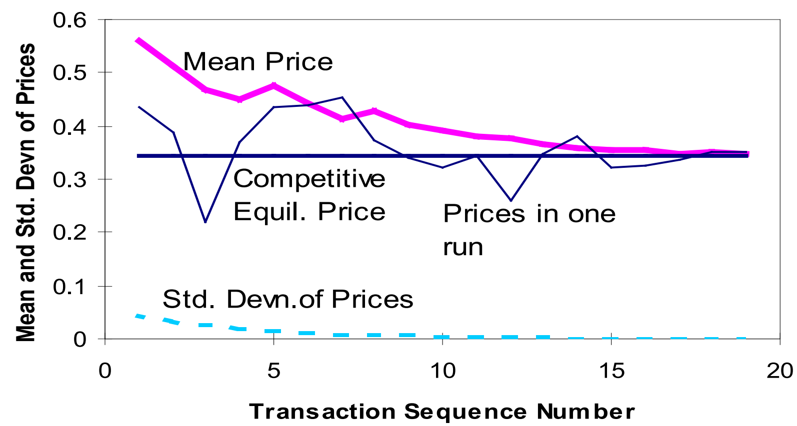
Economy 2



Economy 3



Economy 4



In Conclusion

- Double auction in the pure exchange economy under classical conditions yields Pareto efficient allocations

- The allocations remain efficient even with “zero-intelligence” or minimally rational traders (not offering or accepting exchanges that will make them worse off).

- Simple heuristics yield convergence to the contract curve.