Market structure or agent rationality: How efficiency trades with belief updating?

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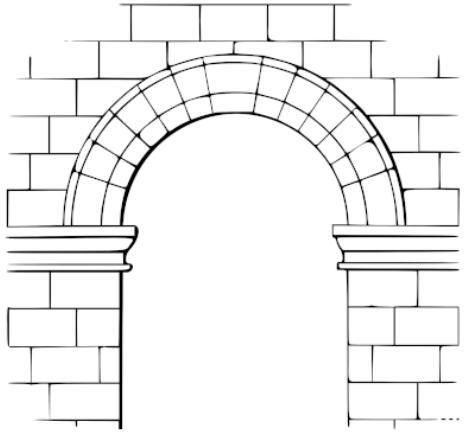
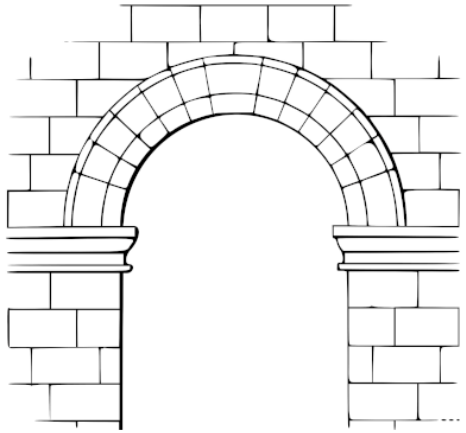
zero intelligence gate



Wilderness of bounded rationality

full rationality gate



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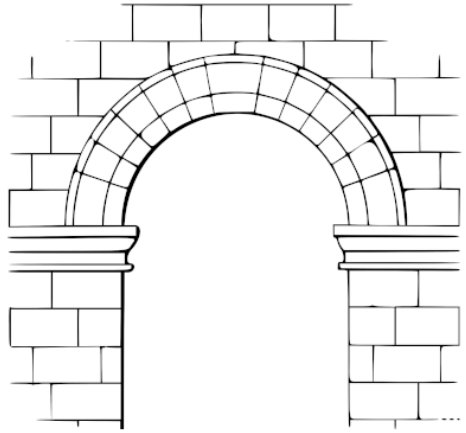

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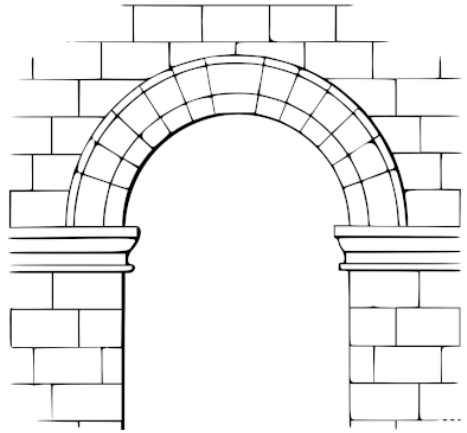

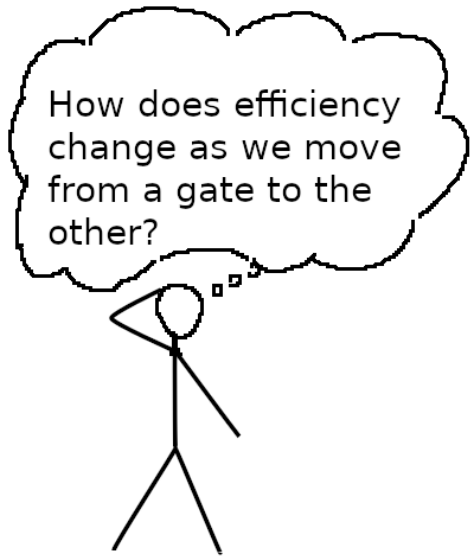

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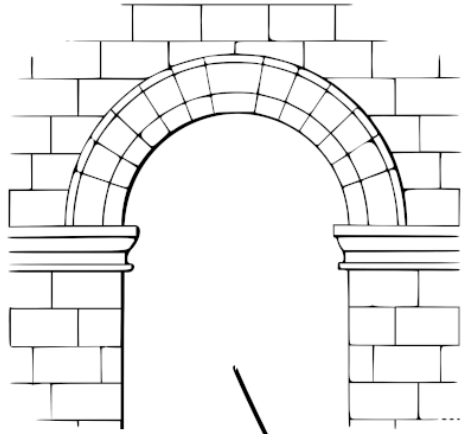

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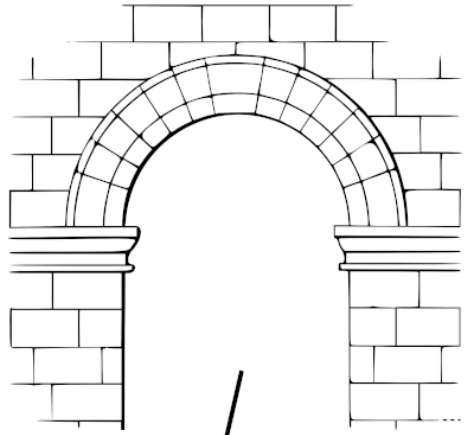
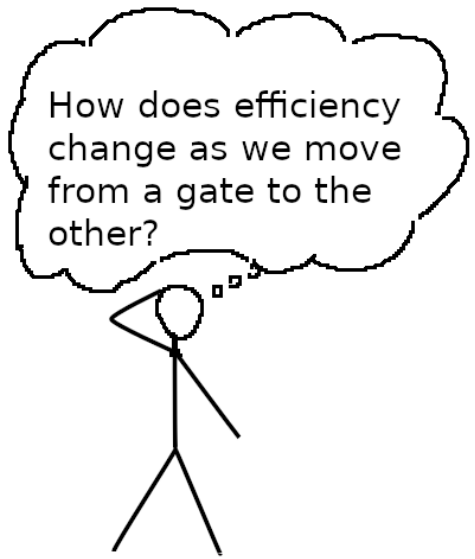




LEARNING

Selection: learning accuracy vs market structure

- Bayesian learning is the ***rationa****l* way of updating beliefs: a **Bayesian** is eventually as accurate as the **best model** (Berk 1966);

- In a competitive setting, the agents who make the **best predictions** are eventually rich and **set prices** (Sandroni 2000);

- Hence, supposing that no one can learn the truth, moving toward Bayesian learning (**rationality**) implies a better (**more efficient**) pricing?

- **Fehr and Tyran** (2005): lack of individual rationality is bad.
- **Gode and Sunder** (1993), **Giachini** (2021): market structure can compensate.

- Our findings: a non-linear **U-shaped relation** between **rationality in learning** and **quality of pricing** (informative efficiency);
- **best outcome** with **ZI agents**!

Modelling the market

Discrete time repeated prediction market model as in Kets et al. (2014) and Bottazzi and Giachini (2019):

- A binary event $s_t \in \{0,1\}$ occurs with $\Pr\{s_t = 1\} = \pi^* \in (0,1)$;
- There are N heterogeneous ***Kelly traders***: they "bet their beliefs";
- Each agent i does not know π^* but has two guesses (models): one optimistic and one pessimistic;
- A model is a probability distribution $(\pi, 1-\pi)$ on s_t: $\left(\pi_o^i, 1-\pi_o^i\right)$ is optimistic and $\left(\pi_p^i, 1-\pi_p^i\right)$ is pessimistic, $0 < \pi_p^i < \pi^* < \pi_o^i < 1$;
- Agents try to learn the true probability adopting a *Bayesian approach* with *limited memory*.

Learning protocol

- The belief (prediction) of agent i for the occurrence of the event at time t is

$$\pi_t^i = w_{t-1}^{i,n}\pi_o^i + \left(1 - w_{t-1}^{i,n}\right)\pi_p^i$$

- n is number of previous observations used to update priors in a Bayesian fashion:
  - if $t \leq n$

$$w_t^{i,n} = w_{t-1}^{i,n}\left(s_t\frac{\pi_o^i}{\pi_t^i} + (1 - s_t)\frac{1-\pi_o^i}{1-\pi_t^i}\right)$$

  - if $t > n$

$$w_t^{i,n} = w_0^{i,n}\prod_{\tau=t-n+1}^{t}\left(s_\tau\frac{\pi_o^i}{\pi_\tau^i} + (1 - s_\tau)\frac{1-\pi_o^i}{1-\pi_\tau^i}\right)$$

  - if $n = 0$

$$w_t^{i,n} = w_0^{i,n}$$

  - w. l. o. g. we assume $w_0^{i,n} = 0.5 \quad \forall i, n$.

Rationality in learning

- n can be considered as a measure of rationality in the learning process:
  - if $n = 0$ we have *Zero Intelligence* learners, $\pi_t^i = \left(\pi_o^i + \pi_p^i\right)/2 \ \ \forall t$;
  - if $n = +\infty$ we have *Bayesian Learning*, the cornerstone of rational learning;
  - if $0 < n < +\infty$, as n grows the agents use an increasing number of past observations to update their beliefs and approach the Bayesian (full information) limit.

Relative entropy

- Given a distribution $(\pi, 1-\pi)$, its *relative entropy* with respect to the true distribution $(\pi^*, 1-\pi^*)$ is defined as

$$D(\pi^* || \pi) = \pi^* \ln \frac{\pi^*}{\pi} + (1-\pi^*) \ln \frac{1-\pi^*}{1-\pi};$$

- it measures the amount of information lost when approximating $(\pi^*, 1-\pi^*)$ with $(\pi, 1-\pi)$: measure of "how different" a distribution is w. r. t. another.

- In a dynamic (online) learning setting, $\{(\pi_t, 1-\pi_t),\ t = 1, 2, \ldots\}$, one can use its (infinite) average

$$\overline{D}(\pi^* || \pi) = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} \left(\pi^* \ln \frac{\pi^*}{\pi_\tau} + (1-\pi^*) \ln \frac{1-\pi^*}{1-\pi_\tau} \right)$$

Price informative efficiency and Market Selection

- In a prediction market, prices can be thought as probability distributions (Arrow et al., 2008; Blume and Easley, 2009; Kets et al., 2014; Dindo and Massari, 2020);
- hence, our measure of informative efficiency is the negative of the average relative entropy of prices $\{(P_{t,}1-P_t), t=1,2,\ldots\}$:

$$\mathcal{E} = -\overline{D}(\pi^* || P)$$

- in the long-run only one agent generically survives: the one with lowest average relative entropy (Blume and Easley, 2009; Kets et al., 2014);
- thus, prices converge to the *most accurate* (lowest avg. relative entropy) beliefs in the markets: calling i^* the most accurate agent it is

$$P_t \to \pi_t^{i^*}$$

- it follows that the price informative efficiency of a prediction market characterized by a rationality level n is

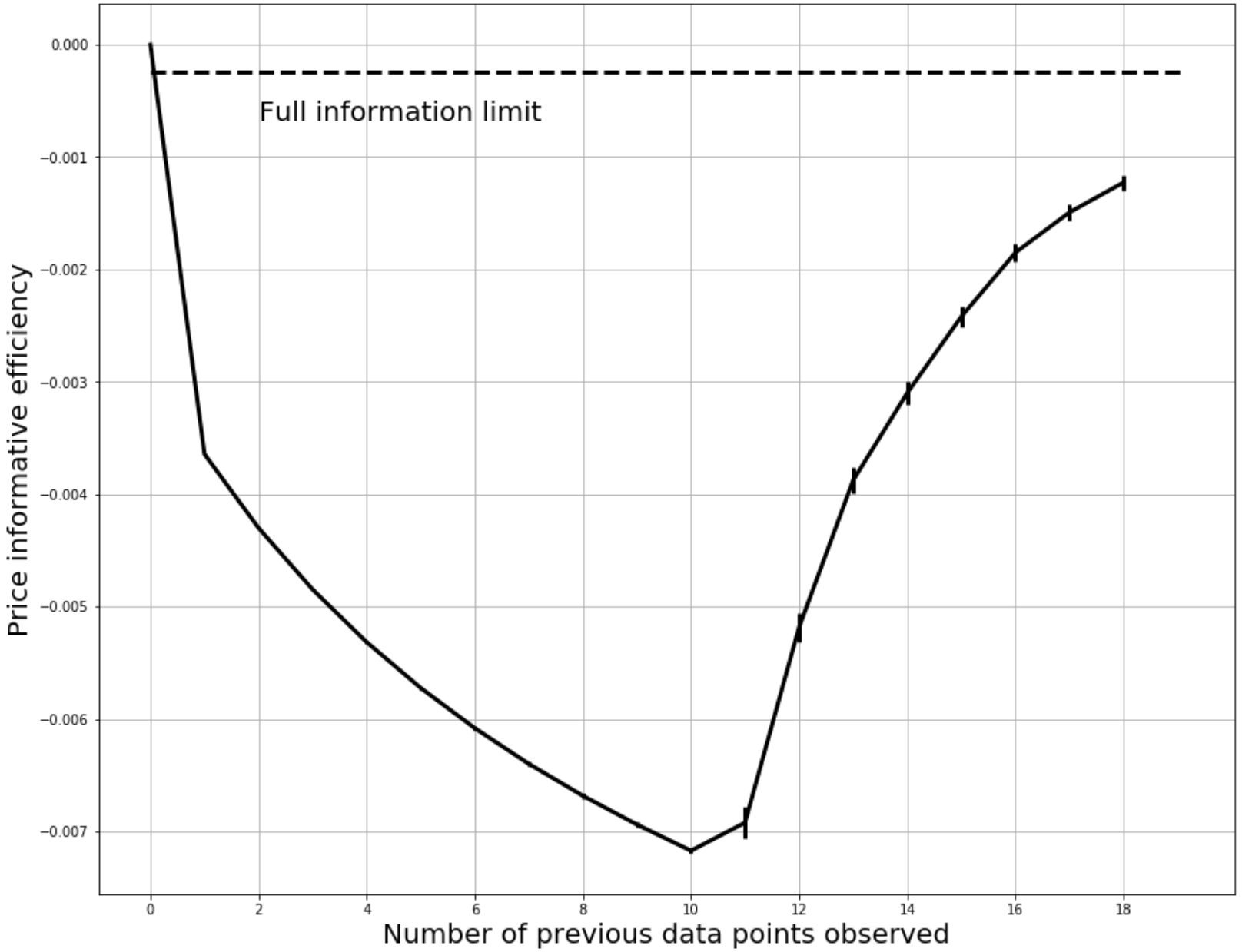
$$\mathcal{E}^n = -\overline{D}\left(\pi^* || \pi^{i^*,n}\right)$$

Results

Price Informative Efficiency as the number of previous observations used for learning increases

$$\pi^* = 0.65$$

$$N = 50$$



Results

$\pi^* = 0.65$

$N = 50$

N	Price Info. Efficiency	Var. wrt Bayes.	Models i^*	Av. Belief i^*	Var. wrt π^*
0	-1.92e-06 +\- 0.0	99.2 %	0.349,0.949	0.649 +\- 0.0	0.2 %
1	-0.00364446 +\- 1.16e-06	-1377.7 %	0.588,0.783	0.474 +\- 0.002	27.1 %
2	-0.00430069 +\- 2.45e-06	-1643.7 %	0.588,0.783	0.54 +\- 0.002	16.9 %
3	-0.00484812 +\- 3.74e-06	-1865.7 %	0.588,0.783	0.573 +\- 0.002	11.9 %
4	-0.005317 +\- 4.97e-06	-2055.8 %	0.588,0.783	0.597 +\- 0.002	8.2 %
5	-0.00572448 +\- 6.14e-06	-2221.0 %	0.588,0.783	0.616 +\- 0.002	5.2 %
6	-0.00608179 +\- 7.26e-06	-2365.9 %	0.588,0.783	0.628 +\- 0.002	3.4 %
7	-0.00639958 +\- 8.29e-06	-2494.8 %	0.588,0.783	0.637 +\- 0.002	2.1 %
8	-0.00668374 +\- 9.27e-06	-2610.0 %	0.588,0.783	0.644 +\- 0.002	1.0 %
9	-0.00693922 +\- 1.021e-05	-2713.6 %	0.588,0.783	0.648 +\- 0.001	0.2 %
10	-0.00717029 +\- 1.111e-05	-2807.3 %	0.06,0.661	0.652 +\- 0.001	0.3 %
11	-0.00682258 +\- 6.72e-05	-2666.3 %	0.06,0.661	0.654 +\- 0.001	0.6 %
12	-0.00507953 +\- 6.106e-05	-1959.5 %	0.06,0.661	0.656 +\- 0.001	0.9 %
13	-0.00378812 +\- 5.283e-05	-1435.9 %	0.06,0.661	0.657 +\- 0.001	1.1 %
14	-0.00301785 +\- 4.667e-05	-1123.6 %	0.06,0.661	0.658 +\- 0.001	1.3 %
15	-0.00234045 +\- 4.28e-05	-849.0 %	0.06,0.661	0.659 +\- 0.0	1.4 %
16	-0.00178504 +\- 3.756e-05	-623.8 %	0.06,0.661	0.659 +\- 0.0	1.4 %
17	-0.00143462 +\- 3.259e-05	-481.7 %	0.06,0.661	0.66 +\- 0.0	1.5 %
18	-0.00117253 +\- 2.945e-05	-375.4 %	0.06,0.661	0.66 +\- 0.0	1.5 %
...	...	...	...	...	...
+oo	-0.00024663 +\- 0.0	0.0 %	0.06,0.661	0.661 +\- 0.0	1.6 %

Concluding remarks

- We investigate the relationship between **rationality in Bayesian learning** and market's **informative efficiency**;

- Our results highlight a **U-shaped relationship** with the maximum efficiency reached with ZI agents:
  - two forces at work: **selection** operated by the **market** (structure) Vs. **selection** operated by **individual agents** (learning);
  - forces **misaligned**: decreasing part of the U;
  - forces **aligned**: increasing part of the U;
  - ***structure works better with ZI agents***.

- References, suggestions, indications are welcomed!

- Next: extend and (possibly) generalize the framework.

References

Arrow, K. J., Forsythe, R., Gorham, M., Hahn, R., Hanson, R., Ledyard, J. O., ... & Zitzewitz, E. (2008). The promise of prediction markets. *Science*, 320(5878), 877-878.

Berk, R. H. (1966). Limiting behavior of posterior distributions when the model is incorrect. *The Annals of Mathematical Statistics*, 51-58.

Blume, L., & Easley, D. (2009). The market organism: long-run survival in markets with heterogeneous traders. *Journal of Economic Dynamics and Control*, *33*(5), 1023-1035.

Bottazzi, G., & Giachini, D. (2019). Far from the madding crowd: Collective wisdom in prediction markets. Quantitative Finance, 19(9), 1461-1471.

Fehr, E., & Tyran, J. R. (2005). Individual irrationality and aggregate outcomes. *Journal of Economic Perspectives*, *19*(4), 43-66.

Kets, W., Pennock, D. M., Sethi, R., & Shah, N. (2014). Betting strategies, market selection, and the wisdom of crowds. In *Twenty-Eighth AAAI Conference on Artificial Intelligence*.

Giachini, D. (2021). Rationality and asset prices under belief heterogeneity. *Journal of Evolutionary Economics*, *31*(1), 207-233.

References

Gode, D. K., & Sunder, S. (1993). Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of political economy*, *101*(1), 119-137.

Sandroni, A. (2000). Do markets favor agents able to make accurate predictions?. *Econometrica*, *68*(6), 1303-1341.